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Mechanics and strength of materials

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Lecture program

1. Kinematics of a point.
2. Motion of a rigid body
3. Dynamics of free and constrained motion of a point
4. Dynamics of a rigid body
5. Conservation laws.
6. Work, power and kinetic energy
7. Mass geometry and impact theory
8. Tension and compression. Hooke's law.
9. Bending.
10. Bending line of beam.
11. Shear, torsion and buckling.
12. Hypothesis of exertion. Combined stress.
13. Energy methods

Literature

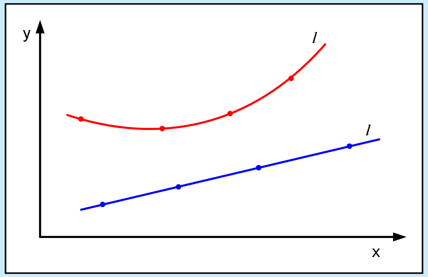
1. **TAYLOR J., Classical mechanics,** University Science Books, 2005
2. **SCHECK F., Mechanics** - From Newton's Laws to Deterministic Chaos, Springer, 2003
3. **SINGH U.K., DWIVEDI M., Problems and solutions in mechanical engineering,** New Age International, 2007
4. **Mott R., Applied strength of materials,** Upper Saddle River, Pearson/Prentice Hall, 2008.
5. **SIUTA W., Mechanika Techniczna,** Wydawnictwa Szkolne i Pedagogiczne, Warszawa 1985.
6. **MISIAK J., Mechanika Ogólna,** WNT, Warszawa 1998.
7. **ŻUCHOWSKI R., Wytrzymałość materiałów,** Oficyna Wydawnicza PWr., Wrocław 1998.

Introduction

- **KINEMATICS – section describing the mechanics of motion of a point or block, without taking into account the weight and causes of change in motion - The geometry of motion**
- **MOTION – described as changing body position relative to the reference body which remains at rest**

Track of a point

This is a solid line *l* formed by the subsequent location of a moving point. Point path may be a straight line or any curve



Kinematic equations of motion of a material point

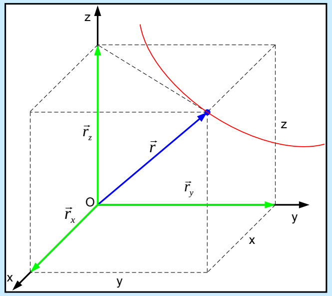
Rectangular coordinates

$x = f_1(t),\quad y = f_2(t),\quad z = f_3(t)$

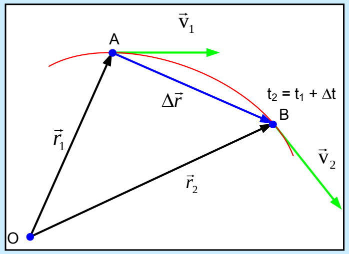
Radius – vector

$$\vec{r} = \vec{r}(t)$$

$r_x = x(t),\quad r_y = y(t),\quad r_z = z(t)$



Velocity of a point



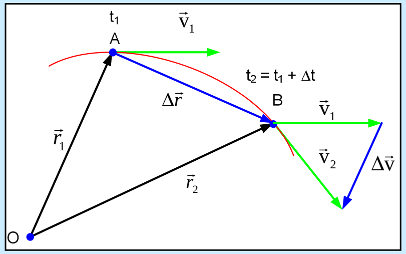
Average velocity

$$\vec{v}_{sr} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t}$$

Instantaneous velocity

$$\vec{v} = \lim_{\Delta t \to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \dot{\vec{r}}$$

Acceleration of a point



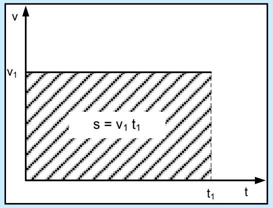
Medium Acceleration

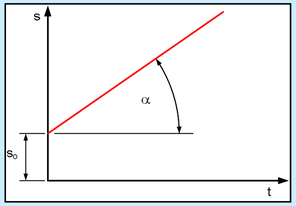
$$\vec{a}_{sr} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}$$

Instantaneous Acceleration

$$\vec{a} = \lim_{\Delta t \to 0}\frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \dot{\vec{v}} = \ddot{\vec{r}}$$

The equations of uniform linear motion





velocity $v = \frac{ds}{dt} = const$

$tg\ \alpha = \frac{s}{t}$

distance $s = s_o + vt$

$tg\ \alpha = v$

Equations of linear motion variable uniformly

acceleration $a = \frac{dv}{dt} = const$

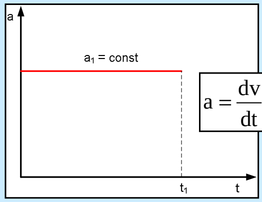
velocity $v = v_o + at$

distance $s = s_o + v_o t + \frac{at^2}{2}$

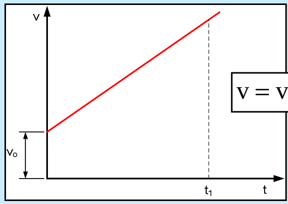
a > 0 uniformly accelerated motion

a < 0 uniformly retarded motion

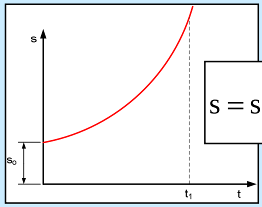
Linear motion variable uniformly



$a = \frac{dv}{dt} = const$

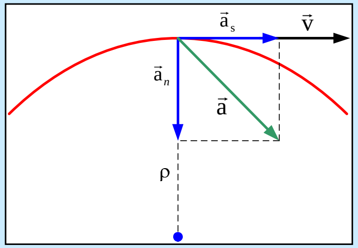


$v = v_o + at$



$s = s_o + v_o t + \frac{at^2}{2}$

Curvilinear motion



Tangential acceleration

$a_t = \dot{v} = \frac{dv}{dt}$

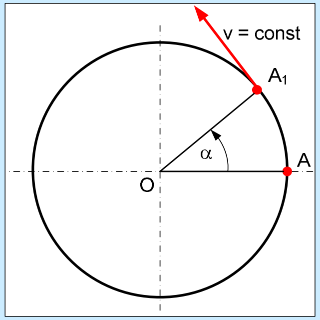
Normal Acceleration

$a_n = \frac{v^2}{\rho}$

$\vec{a} = \vec{a}_t + \vec{a}_n$

value $a = \sqrt{a_n^2 + a_t^2}$

Uniform motion in a circle



distance $s = r\alpha$

linear velocity $v = \frac{ds}{dt} = \frac{d(r\alpha)}{dt} = \omega r$

angular velocity $\omega = \frac{d\alpha}{dt}$

$$\omega = 2\pi \frac{n}{60}$$

Angular acceleration

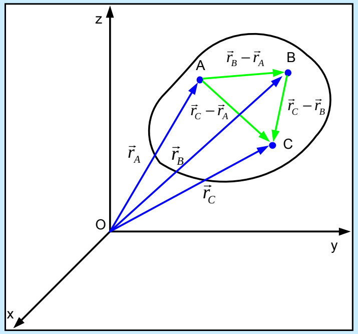
Tangential acceleration $a_s = \frac{dv}{dt} = \frac{d(r\omega)}{dt} = r\frac{d\omega}{dt} = r\varepsilon$

$$\varepsilon = \frac{d\omega}{dt}$$

Normal Acceleration $a_n = \frac{v^2}{r} = \omega^2 r$

$$a = \sqrt{a_s^2 + a_n^2} = \sqrt{(\omega^2 r)^2 + (\varepsilon r)^2} = r\sqrt{\omega^4 + \varepsilon^2}$$

Motion of a rigid body

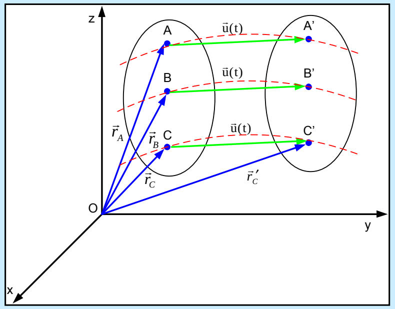


Rigid body – distances between points are unchanged

Motion of a rigid body can be determined by vector equations of three points A, B, C

$$\vec{r}_A = \vec{r}_A(t) \qquad \vec{r}_B = \vec{r}_B(t) \qquad \vec{r}_C = \vec{r}_C(t)$$

Motion of a rigid body



$$\vec{r}_A'(t) = \vec{r}_A(t_o) + \vec{u}(t),$$

$$\vec{r}_B'(t) = \vec{r}_B(t_o) + \vec{u}(t),$$

$$\vec{r}_C'(t) = \vec{r}_C(t_o) + \vec{u}(t),$$

$\vec{u}(t)$ - Shift is equal for all points of the body

Motion of a rigid body

Differentiating the above equations of motion vectors with respect to time we get velocity and acceleration of points A,B,C

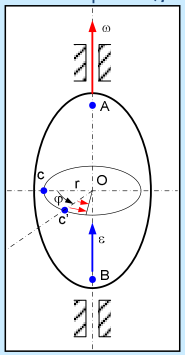
$$\vec{v}_A = \vec{v}_B = \vec{v}_C = \frac{d\vec{u}(t)}{dt}$$

$$\vec{a}_A = \vec{a}_B = \vec{a}_C = \frac{d^2\vec{u}(t)}{dt^2}$$

Vectors of velocity and acceleration of all points of a rigid body, moving with an advancing motion are the same at the same time

The rotary motion of the solid around a fixed axis

A lump can be rotated around the axis only (passing through two points), called the axis of rotation



$$\varphi = \varphi(t)$$

$$\omega = \dot{\varphi}$$

$$\varepsilon = \dot{\omega} = \ddot{\varphi}$$

$$v = \omega r$$

$$\vec{v} = \vec{\omega} \times \vec{r}$$

Acceleration in rotation

Acceleration of tangential and normal acceleration of any point of a rigid body lying at a distance r from the axis of rotation with respect to time we get by differentiating the formula for linear velocity yielding :

$$a_t = \dot{v} = r\frac{d\omega}{dt} = r\varepsilon$$

$$a_n = \frac{v^2}{r} = \frac{r^2\omega^2}{r} = r\omega^2$$

$$a = \sqrt{a_t^2 + a_n^2} = r\sqrt{\varepsilon^2 + \omega^4}$$

Newton's Second Rule

The change of motion is proportional to the applied force and takes place in the direction of the straight line along which that force acts

$$\vec{F} = \frac{d}{dt}(m\vec{v})$$

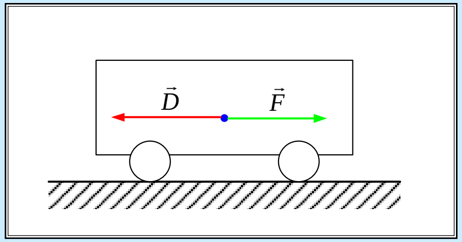
$m = \text{const}$

$$m\frac{d\vec{v}}{dt} = m\vec{a} = \vec{F}$$

Inertia

Cart accelerates with the acceleration $\vec{a}$. We must therefore work force $\vec{F}=m\vec{a}$. According to the principle of action and reaction in our hands, work the same force from the cart, but returned to the contrary $\vec{D}=-m\vec{a}$

This is the force of inertia (D'Alembert)



The dynamics of free and constrained motion of a point. D'Alembert's Principle

In the case of free motion of a point system of active forces balances the force of inertia

$$\sum \vec{F}_i + (-m\vec{a}) = 0$$

In the case of motion constrained point force response of active and balanced ties with the force of inertia

$$\sum \vec{F}_i + \sum \vec{R}_i + (-m\vec{a}) = 0$$

The momentum of the material point

Write the Newton's second law in the form of:

$$\frac{d}{dt}(m\vec{v}) = \frac{d\vec{p}}{dt} = \sum \vec{F}_i$$

The vector is called the momentum or quantity of motion of a point .

$$m\vec{v} = \vec{p}$$

The principle of conservation of momentum of a point

In the event that a material point does not work force or forces in balance, the momentum of the material point is constant .

$$\frac{d}{dt}(m\vec{v}) = \frac{d\vec{p}}{dt} = 0$$

$$\vec{p} = \text{const.}$$

The principle of mass momentum and impulse forces

The Newton's second law:

$$\vec{F}dt = d(m\vec{v})$$

or

$$\vec{F}dt = d\vec{\Pi}$$

Elementary impulse force acting on a material point is equal to the gain in momentum of the elementary point .

The principle of mass momentum and impulse forces

Integrating both sides of previous equation, we obtain

$$\int_{t_1}^{t_2} \vec{F}dt = (m\vec{v}_2 - m\vec{v}_1)$$

$$\vec{\Pi} = \int_{t_1}^{t_2} \vec{F}dt$$

- impulse is the total force F in the time interval t_2-t_1 ,

We obtain

$$\vec{\Pi} = \vec{p}_2 - \vec{p}_1$$

The growth momentum of a moving mass point is equal to the total impulse forces

Angular momentum of material point

$$\vec{K}^O = \vec{r} \times m\vec{v}$$

This is the moment of momentum by the chosen pole

$$\dot{\vec{K}}^O = \vec{M}^O$$

$$\vec{M}^o = 0 \Rightarrow \vec{K}^o = const$$

Dynamic equations of motion of a material point

$$\vec{F} = m\vec{a}$$

Dynamic equation of motion in vector form can be replaced by three analytical equations :

$$F_x = m\ddot{x} \quad , \quad F_y = m\ddot{y} \quad , \quad F_z = m\ddot{z}$$

1. The first task (simple) - these are the parametric equations of the track, which moves the material point, find the force acting on point

2. The second task (reverse) - to determine the equation of motion, with a particular strength

Relative motion of a material point

Motion relative to the fixed point is defined by the equation

$$m\vec{a}_b = \sum \vec{F}_i$$

and

$$\vec{a}_b = \vec{a}_w + \vec{a}_u$$

In a moving system the equation of motion is determined

$$m\vec{a}_w = \sum \vec{F}_i - m\vec{a}_u$$

In whitch $\vec{D}_u = -m\vec{a}_u$ called the lifting force of inertia . It is equal to mass multiplied by acceleration of floating point and is opposite than $\vec{a}_u$

Work of a constant force

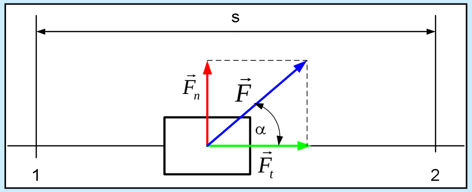
Permanent work of a force on a straight movement in the direction of force is called the product of this force by the length of shifts

$$A = Fs$$

The unit of work in the SI system is J (dżul):

$$J = Nm = \frac{kgm}{s^2} m$$

Work of a constant force



If the vector of force is inclined to the direction in terms of a shift, the work is calculated from the formula:

$$A = F_t s = Fs \cos \alpha$$

The work makes only a tangential force component to the trajectory of F_t. Jobs normal component to the trajectory of F_n is equal to zero

Work of a constant force

$\alpha = 0°$	$A = Fs > 0$
$0° < \alpha < 90°$	$A = Fs \cos \alpha > 0$
$\alpha = 90°$	$A = 0$
$90° < \alpha < 180°$	$A = Fs \cos \alpha < 0$
$\alpha = 180°$	$A = -Fs < 0$

In general :
a) work is a scalar ,

b) work may take positive or negative values and zero,

c) work is done only by component of force tangential to the path.

Work of a variable force

The elementary work of the variable force on a shift $d\vec{s}$ is called the dot product of force $\vec{F}$ by an elementary shift

$$\delta A = \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}}$$

because

$$\vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} = F ds \cos\left(\vec{\mathbf{F}}, d\vec{\mathbf{s}}\right) = F_t ds$$

so

$$\delta A = F_t ds$$

Work of a force on a shift is equal to the total work forces in the respective constituent components of displacements

Work of a variable force

Total work of the position 1 to position 2 on the track are obtained by integrating the expression of an elementary work .

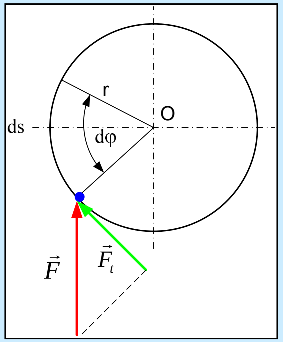
$$A_{1-2} = \int_{s_1}^{s_2} \delta A$$

$$\vec{F} = \vec{i}X + \vec{j}Y + \vec{k}Z \qquad d\vec{s} = \vec{i}dx + \vec{j}dy + \vec{k}dz$$

$$A_{1-2} = \int_{x_1}^{x_2} Xdx + \int_{y_1}^{y_2} Ydy + \int_{z_1}^{z_2} Zdz$$

Work on circular path

When the force $\vec{\mathbf{F}}$ acting on a point moving along a circular path (belt tension belt drive), we obtain



$$\delta A = \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} = F_t ds$$

$$ds = r\, d\varphi$$

$$\delta A = F_t r\, d\varphi$$

Work on circular path

Expression $F_t r$ determines the moment of force $\vec{\mathbf{F}}$ relative measure O (eg, center of the disc). We call it torque

$$M_o = F_t r$$

The formula for the elementary work takes the form:

$$\delta A = M_o\, d\varphi$$

Work on circular path

Total work on the road φ_1 to φ_2 determine the angular integral

$$A_{1-2} = \int_{\varphi_1}^{\varphi_2} M_o \, d\varphi$$

Power

In practice, we are often interested in the volume of work the motor or the machine can perform per unit time. The work is related to the unit of time dt is called power. In particular, the instantaneous power of the employment relationship is called the elementary to the time at which this work was performed

$$P = \frac{\delta A}{dt}$$

expression for the instantaneous power is presented in the following form:

$$P = \frac{F_t ds}{dt}$$

or

$$P = F_t v$$

Power in rotation

Power

$$P = \frac{M_o d\varphi}{dt}$$

Angular velocity

$$\frac{d\varphi}{dt} = \omega$$

Power in rotation

$$P = M_o \omega$$

Power

When the rotation velocity setpoint is using the engine speed n, rpm - then the angular velocity is calculated from the formula :

$$\omega = \frac{2\pi n}{60}$$

Power

$$P = M_o \frac{\pi n}{30}$$

The fundamental unit of power is *W = J/s = Nm/s*

This technical units : *kW i MW*

Efficiency

Efficiency is the ratio of received work (power) to operate the input work (power)

Mechanical efficiency ratio determined by:

$$\eta = \frac{A_u}{A_0} = \frac{P_u}{P_0}$$

The efficiency is a dimensionless number and can be regarded as a characteristic measure of comparison engines and machines, as far as the economical way to work in utilization is not loaded

Principle of work and kinetic energy

$$F_t = ma_t = m\frac{dv}{dt}$$

$$\delta A = F_t ds$$

$$\delta A = m\frac{dv}{dt}ds = mvdv$$

$$ds = vdt$$

The right side of this equation is a function of the total differential *called the kinetic energy of a moving material point* .

$$E_k = \frac{mv^2}{2}$$

Principle of work and kinetic energy

Based on the above dependence we obtain

$$\delta A = dE$$

This equation shows the principle of work and kinetic energy, expressed in the form of differential equation.
After integration we obtain

$$A_{1-2} = E_2 - E_1$$

The kinetic energy of a moving material point increases or decreases the volume of work done by forces acting on the material point .

Function of field forces

Total differential field function is equal:

$$d\Phi = \frac{\partial \Phi}{\partial x}dx + \frac{\partial \Phi}{\partial y}dy + \frac{\partial \Phi}{\partial z}dz$$

So the differential was equal to the elementary work

$$\delta A = Xdx + Ydy + Zdz$$

must be met depending on

$$X = \frac{\partial \Phi}{\partial x} \qquad Y = \frac{\partial \Phi}{\partial y} \qquad Z = \frac{\partial \Phi}{\partial z}$$

Potential of field forces

Vector of field force can be written in the form

$$\vec{F} = \frac{\partial \Phi}{\partial x}\vec{i} + \frac{\partial \Phi}{\partial y}\vec{j} + \frac{\partial \Phi}{\partial z}\vec{k}$$

The right side is the gradient of the function, Φ so

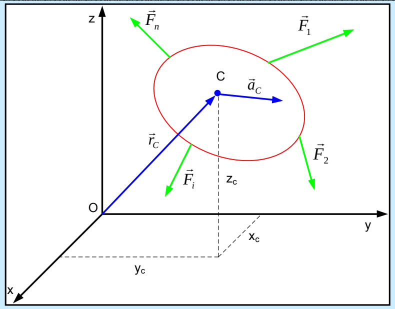
$$\vec{F} = \mathrm{grad}\Phi$$

Progressive motion of a rigid body

$$m\vec{a}_c = \sum_{i=1}^{n} \vec{F}_i$$

where: *m* – mass of a rigid body

$\vec{a}_C$ - acceleration of center of mass



$$m\ddot{x}_c = \sum_{i=1}^{n} F_{ix} \qquad m\ddot{y}_c = \sum_{i=1}^{n} F_{iy} \qquad m\ddot{z}_c = \sum_{i=1}^{n} F_{iz}$$

The theorem on the derivative of angular momentum

The derivative of angular momentum of the body relative to its center of mass is equal to the geometric moments of all forces external to this measure

$$\frac{d\vec{K}_c}{dt} = \sum_{i=1}^{n} \vec{M}_{ic}$$

Progressive motion of a rigid body

The progressive motion of all points of a rigid body have the same velocity, such as the center of mass of the body. Thus, Angular momentum of a rigid body relative to the center of mass is equal to zero $\vec{K}_c = 0$

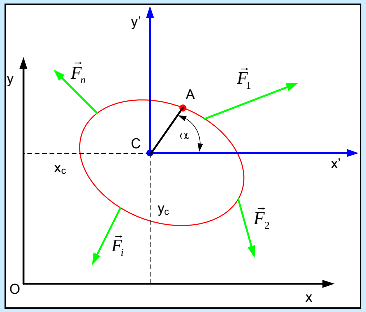
The equation shows that when the body moves with an advancing motion is the sum of the geometric moments of external forces to the center of body mass must be zero

$$\sum_{i=1}^{n} \vec{M}_{ic} = 0$$

The external forces must create a layout that has the result $\vec{W}$ of a line of action passing through the center of mass C.

Plane motion of rigid body

Shown the drawing section of the body obtained by intersection of the plane parallel to the plane of the directing and passing through the center of mass C



Dynamic equations of motion of a rigid body

To obtain the dynamic equations of motion of a rigid body

we use a flat :

- dynamic equations of Progressive motion
- the principle of angular momentum in a rotating motion

$$I_z \varepsilon = M_z$$

Dynamic equations of motion of a rigid body

The equation of progressive motion in the x direction → $$m\ddot{x}_c = \sum_{i=1}^{n} F_{ix}$$

The equation of progressive motion in the y direction → $$m\ddot{y}_c = \sum_{i=1}^{n} F_{iy}$$

The principle of conservation of angular momentum in a rotating motion → $$I_z \varepsilon = \sum_{i=1}^{n} M_{iz}$$

$\ddot{x}_c, \ddot{y}_c$ – hardware acceleration of the center of mass C

I_z – moment of inertia with respect to the axis z of the body

ε – angular acceleration of the axis z of rotation of the body

Moment of inertia

Moment of inertia of a material point relative to the plane, an axis or pole is product of the mass by the square of the distance of this point from the plane, an axis or pole :

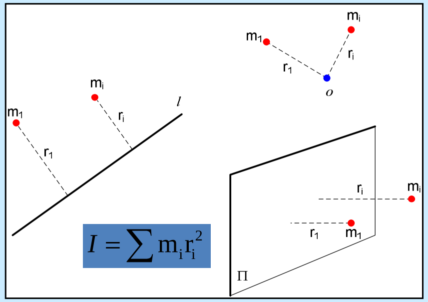
$$I = mr^2$$

Unit is

$$[\ I\ ] = \text{kg m}^2$$

Moment of inertia of material points system

Moment of inertia of material points system the plane, an axis or pole is called the sum of the moments of inertia of all the material points of this plane, axis or pole.



Moment of inertia of the constant

Moment of inertia of the constant (lines, surfaces or solid material) of view of the plane, called the axis or pole of the integral

$$I = \int r^2 dm$$

stretched to the whole mass of the system

Moment of inertia of the plane

Moments of inertia with respect to the coordinate planes define the formulas:

$$\left.\begin{aligned} I_{xy} &= \sum m_i z_i^2, \\ I_{yz} &= \sum m_i x_i^2, \\ I_{zx} &= \sum m_i y_i^2 \end{aligned}\right\}$$

Moment of inertia axis and pole

Moment of inertia axis

$$\left.\begin{aligned} I_x &= \sum m_i \left(y_i^2 + z_i^2\right), \\ I_y &= \sum m_i \left(x_i^2 + z_i^2\right), \\ I_z &= \sum m_i \left(x_i^2 + y_i^2\right) \end{aligned}\right\}$$

Moment of inertia pole

$$I_o = \sum m_i \left(x_i^2 + y_i^2 + z_i^2\right)$$

Moment of deviation

The moment of deviation of a point mutually perpendicular planes, called the product of the mass by the distance from the point of planes

$$D_{\alpha\beta} = m\ r_1 \rho_1$$

Moments of deviation can be positive, negative and, in particular, equal to zero

Moment of deviation

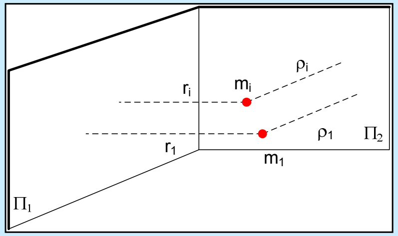
Moment of deviation of material points relative to the two mutually perpendicular planes, is the sum of the moments of deviation of individual points of the material terms of these planes .

$$D_{\alpha\beta} = \sum m_i\ r_i \rho_i$$

For the constant

$$D_{\alpha\beta} = \int r\rho\ dm$$

extended to the whole mass .



Moment of deviation

The spatial coordinates of the system of material points is three moments of deviation

$$D_{xy} = \sum m_i\ x_i y_i,$$
$$D_{zx} = \sum m_i\ z_i x_i,$$
$$D_{yz} = \sum m_i\ y_i z_i$$

In flat coordinates the material system has one moment of deviation

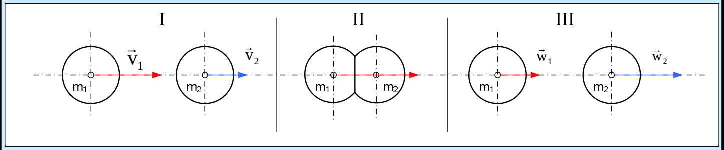
$$D_{xy} = D = \sum m_i\ x_i y_i$$

Parallel transformation of the moments of inertia

Moment of inertia with respect to any axis is equal to the momentum parallel to the axis passing through the center of gravity plus the product of the total weight by the square of the distance of the two axes.

$$I_l = I_c + md^2$$

Central simple collision



$$m_1 v_1 + m_2 v_2 = m_1 w_1 + m_2 w_2$$

At the same time w_1 and w_2 shows the velocity of both masses after the collision. Their appointment also use the equation resulting from energy considerations . Velocity w_1 and w_2 will depend on whether the loss of kinetic energy

Central simple collision

a) energy returned 100% (perfectly elastic collision of bodies)
b) energy absorbed at 100% (perfectly plastic body collision),
c) energy absorbed in part (the actual collision of bodies.)

For the determination of these losses will introduce the so-called energy. collision rate, calling him a model

$$k = \frac{w_2 - w_1}{v_1 - v_2} \qquad 0 \le k \le 1$$

The limit values correspond to the ratio k

$k = 1$ for the body perfectly elastic

$k = 0$ for perfectly plastic body

Central simple collision

The actual loss of kinetic energy is

$$\Delta E_1 = \frac{1}{2}\left[m_1 v_1^2 + m_2 v_2^2 - m_1 w_1^2 - m_2 w_2^2\right]$$

After substitution of equations for w_1 and w_2 receive

$$\Delta E_1 = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}\left(v_1 - v_2\right)^2\left(1 - k^2\right)$$

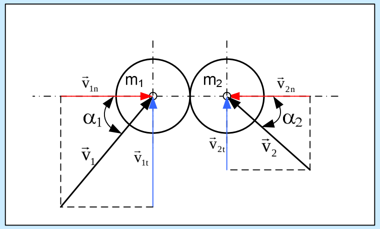
Diagonal central collision

Spread the velocity vectors v_1 and v_2 into components normal and tangential to the plane of contact

$$v_{1n} = v_1 \cos\alpha_1 \qquad v_{2n} = v_2 \cos\alpha_2$$

$$v_{1t} = v_1 \sin\alpha_1 \qquad v_{2t} = v_2 \sin\alpha_2$$

$\vec{v}_{1t} = \vec{w}_{1t}$ and $\vec{v}_{2t} = \vec{w}_{2t}$

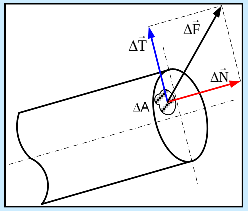


Finally, after collision

$$\left.\begin{aligned} \vec{w}_1 &= \vec{w}_{1n} + \vec{w}_{1t} = \vec{w}_{1n} + \vec{v}_{1t}, \\ \vec{w}_2 &= \vec{w}_{2n} + \vec{w}_{2t} = \vec{w}_{2n} + \vec{v}_{2t} \end{aligned}\right\}$$

Stress

Let's consider the force $\vec{\Delta F}$ per element of area ΔA



Stress $\vec{p}$ at a point where A is called a solid boundary, which seeks the ratio of internal force $\vec{\Delta F}$ by elementary field ΔA of this section, where the field tends to zero.

$$\vec{p} = \lim_{\Delta A \to 0} \frac{\Delta \vec{F}}{\Delta A} = \frac{d\vec{F}}{dA}, \qquad \frac{N}{m^2}$$

Tangential and normal stress

After spreading the force $\vec{\Delta F}$ on the normal component $\vec{\Delta N}$ and tangential $\vec{\Delta T}$ will receive the normal $\vec{\sigma}$ and tangential $\vec{\tau}$ stresses:

$$\vec{\sigma} = \lim_{\Delta A \to 0} \frac{\Delta \vec{N}}{\Delta A} = \frac{d\vec{N}}{dA}, \qquad \frac{N}{m^2}$$

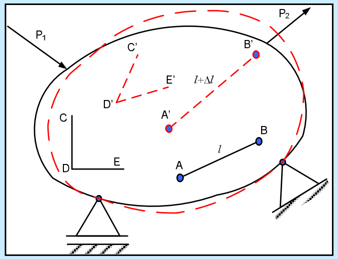
$$\vec{\tau} = \lim_{\Delta A \to 0} \frac{\Delta \vec{T}}{\Delta A} = \frac{d\vec{T}}{dA}, \qquad \frac{N}{m^2}$$

Linear Deformation

Segment AB = l - changed after loading: A'B' = $l+\Delta l$.

The average elongation of the segment AB will be :

$$\varepsilon_{sr} = \frac{\Delta l}{l}$$

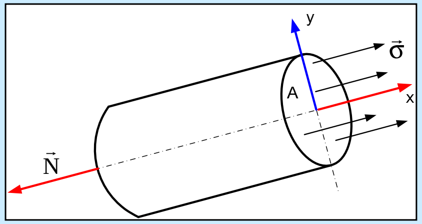


Local extension :

$$\varepsilon = \lim_{\Delta l \to 0} \frac{\Delta l}{l}$$

Simple stretching

Simple stretching occurs when the result of reduction of the internal forces from the center of the cross section of the body will receive only the principal vector, normal to this section



N – axial force ,

A – cross-sectional area of bar

σ – normal stress

Simple stretching

The condition of balance - the sum of the projections of all forces in the direction of the axis of the rod

$$\sum F_{ix} = \int_{(A)} \sigma \, dA - N = 0$$

$$N = \int_{(A)} \sigma \, dA$$

When the stresses are the same for the entire cross-section:

$$N = \sigma A$$

This means that the main vector is equal to the strength of the aggravating and the absence of other stresses on the cross-sectional area A.

Simple stretching

Effort of the material – degree of approximation of the load material into a critical state

The condition of the bar strength :

$$\sigma = \frac{N}{A} \leq \sigma_{dop}$$

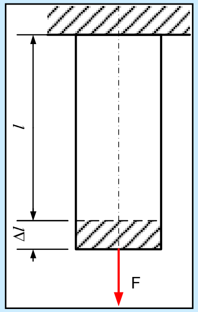
Allowable stress σ_{dop}

$$\sigma_{dop} = \frac{\sigma_{nieb}}{n}$$

n – safety factor ($n \geq 1$),

σ_{nieb} – Dangerous Stress

Hooke's law



$$\Delta l = \frac{F\,l}{EA}$$

$$\varepsilon = \frac{\Delta l}{l} \qquad \sigma = \frac{F}{A}$$

Hooke's law takes the form :

$$\sigma = \varepsilon \, E$$

Lateral deformation

The difference of initial and final thickness is called total stenosis, $\Delta h = h_1 - h$. Ratio of stricture of the total thickness of the initial call to the narrowing of the unit ε_1

$$\varepsilon_1 = \frac{\Delta h}{h}$$

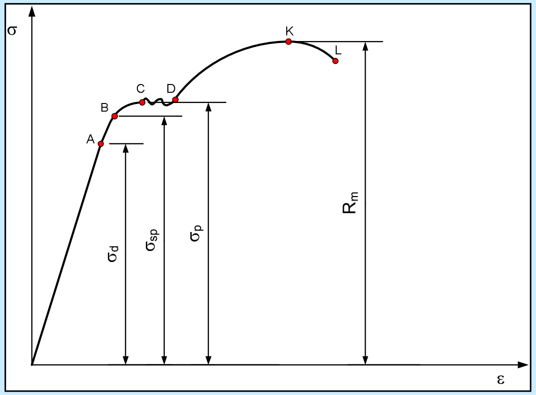
Poisson's number

The absolute value of the ratio of stenosis (swelling) of the unit ε_1 to unit ε elongation (shortening), is called the coefficient of transverse strain and Poisson's number ν

$$\nu = \frac{\varepsilon_1}{\varepsilon}$$

Poisson's number assumes values within the $0 \div 0{,}5$

Graph drawing low carbon steel



Graph drawing low carbon steel

Individual points on the graph means:

A – limit of proportionality (the limit of applicability of Hooke's law)

B – elastic limit - in practice it is assumed that lie near each other, points A and B are of equal value :

$$\sigma_p = \sigma_{sp}$$

C, D - yield Re, - clearly visible on the graph drawing and easy to set only for certain materials, such as low carbon steel.

$$R_e = \frac{F_e}{A_0}$$

Graph drawing low carbon steel

K - yield tensile strength R_m (Emergency strength of the material).

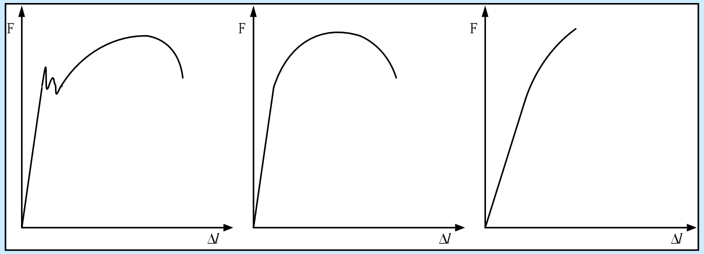
Tensile strength R_m is the ratio of the maximum tensile strength F_{max} obtained in the process of drawing the sample through the box section of the initial sample A_o:

$$R_m = \frac{F_{max}}{A_0}$$

After reaching the stress on the sample arises R_m local constriction (the neck). In place of the sample ruptures - segment KL stretching from the chart.

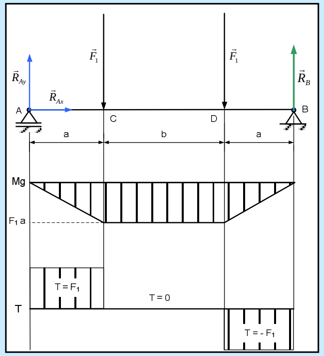
Drawing graphs of different materials

low carbon steel, aluminum alloys | copper | cast



Beam subjected to pure bending

Pure bending can be observed in the case of bending of a prismatic beam loaded as shown



The condition of bending strength

The stresses in the bending rod are the largest in the fibers, thus

$$\sigma_{max} = \frac{M_g}{W_y} \leq \sigma_{dop}$$

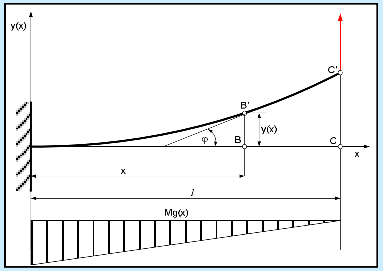
As we approach the fiber axis of the beam load decreases

Beams of equal strength

The condition of constant-strength beam bending strength along the entire length has the form:

$$\sigma = \frac{M_g\,(x)}{W\,(x)} = \sigma_{dop}$$

Deflection of the beam line



Differential equation of deflection of the beam line

$$\frac{d^2y}{dx^2} = \frac{M_g(x)}{E\,I_y}$$

The equation for beam deflection angle

Integrating the previous equation, we obtain the dependence of the variable x is used to designate the angle of deflection j in the form:

$$\varphi(x) = \frac{dy(x)}{dx} = \int \frac{M_g(x)}{E\,I_y}\,dx + C$$

The equation for deflection of the beam line

After re-integration with respect to the variable x we obtain :

$$y = \int \varphi(x)\,dx + D$$

Constants of integration C and D we determine the boundary conditions. We have two types of these conditions and they result from:

- stiffness of the supports (boundary conditions),
- continuity of the beam (continuity conditions).

Shear stress

In cross-section of a tensile rod, tilted at an angle s in addition to normal stresses are also shear stress or shear t:

$$\tau_\alpha = \frac{1}{2}\,\sigma \sin 2\alpha$$

Pure shear

State of stress in these sections, in which there are only a shear stress is called the pure shear

State of pure shear is difficult to be prepared by direct order of the body themselves shear stress, whereas such an effect can be obtained by calling eg, tensile and compressive same, in absolute value σ stress, acting in two mutually perpendicular directions

Technical Shear

A condition of shear strength :

$$\tau = \frac{T}{A} \leq \tau_{dop}$$

where:

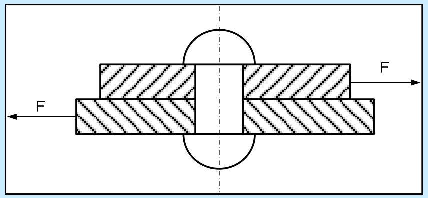
τ_{dop} – allowable shear stress ,

T – shear force,

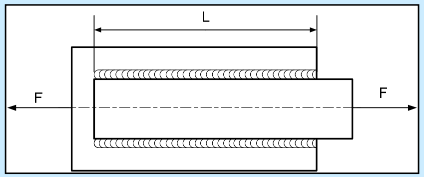
A – cross-section subjected to shear .

Cases of the technical shear

rivets

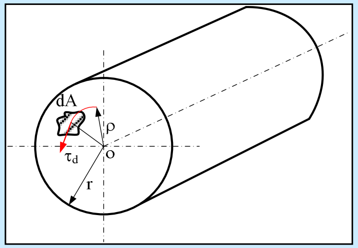


weld



Torque

Elementary torque tension rod axis is :



$$dM = \tau_{\rho} \, dA \, \rho$$

Angle torsion bar

Torsion angle round bar of dimensions *d*, *l* and M_s is the:

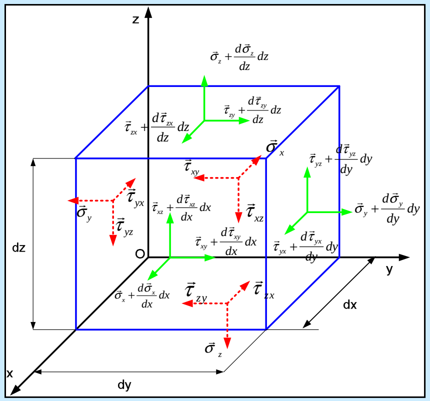
$$\varphi = \frac{M_s\, l}{G\, I_0}$$

Condition of the torsional strength

Stress in the screw rod can not exceed the allowable torsional stress k_s

$$\tau_{max} = \frac{M_s}{W_0} \leq k_s$$

Stresses in the material



Material effort

Material effort – degree of approximation of the load material into the boundary condition .

Hypotheses of material effort – designs for the conversion of the load on the load-dimensional spatial.

Hypotheses effort

Hypotheses effort depending on the adopted measure of effort can be divided into :

- Stress
- Deformation
- Energy
- Mixed

Basic concepts

The purpose of effort hypothesis is to determine the correlation between the components of the effort in a state of stress:

$$W = F\left(\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx}, C_i\right)$$

Critical values of effort can be determined by carrying out the experience for one specific state of stress σ_o, it is best for the uniaxial tension

$$W = F\left(\sigma_0, 0, 0, 0, 0, 0, C_i\right)$$

The hypothesis of the largest tensile stress

The authors of this hypothesis are
Galileusz (1632) and Leibnitz (1864)

According to this hypothesis, a measure of the effort is the largest tensile stress

This is the hypothesis of stress and is occasionally used in fracture mechanics

The hypothesis of the greatest elongation

The authors of this hypothesis are E. Maroitte, B. St Venant and J.V. Poncelet.

The measure of effort according to this hypothesis is the greatest elongation.

$$\varepsilon_1 \le \varepsilon_{kr}$$

$$\varepsilon_2 \le \varepsilon_{kr}$$

$$\varepsilon_3 \le \varepsilon_{kr}$$

For ε_{kr} adopted elongation ε_{zr}, corresponding tensile strength Rm

The hypothesis of the greatest elongation

When the plane state of stress is expressed by means of strain σ_x, σ_y, τ_{xy} the main stress is determined by the formula:

$$\sigma_{1,2} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

and the reduced stress is determined from one of the three equations

$$\sigma_1 - \nu\sigma_2 \le \sigma_{kryt} \qquad \sigma_2 - \nu\sigma_1 \le \sigma_{kryt} \qquad -\nu(\sigma_1 + \sigma_2) \le \sigma_{kryt}$$

With reduced stress should be the left side of the equation, which is higher than the left side of each of the other two equations

The hypothesis of the largest shear stress

Its authors are C.A. Culomb, H. Tresca i J.J. Guest. The measure of effort this hypothesis is the greatest tangential stress

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

In the case of a simple tensile stress is reduced equation has the form:

$$\sigma_{red} = \sigma_{max} - \sigma_{min} \le \sigma_{kryt}$$

$$\sigma_{red} = \sigma_0$$

The hypothesis of the largest shear stress

Where $\sigma_{max} = \sigma_1$ and $\sigma_{min} = \sigma_3$, that:

$$\sigma_{red} = \sigma_1 - \sigma_3 \le \sigma_{kryt}$$

In the case of plane state of stress , when: $\sigma_x = \sigma$, $\sigma_y = 0$, $\tau_{xy} = \tau$, reduced stress can be determined :

$$\sigma_{red} = \sqrt{\sigma^2 + 4\tau^2}$$

The hypothesis of internal energy shear (Huber's hypothesis)

As a measure of effort by this hypothesis is the specific energy shear, which in any state of stress is spatially:

$$\phi = \frac{1+\nu}{6E}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)\right]$$

In the uniaxial state of stress responsible shear energy will be:

$$\phi = \frac{1+\nu}{6E} 2\sigma_0^2$$

The hypothesis of internal energy shear (Huber's hypothesis)

Reduced stress :

$$\sigma_{red} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}$$

Reduced stress in the system of principal stresses can be determined from the equation:

$$\sigma_{red} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

The hypothesis of internal energy shear (Huber's hypothesis)

For a plane state of stress σ_x, σ_y, t_{xy} reduced stress expressed in the equation:

$$\sigma_{red} = \sqrt{(\sigma_x - \sigma_y)^2 + 3\tau^2}$$

If $\sigma_x = \sigma$, $\sigma_y = 0$, $\tau_{xy} = \tau$ this equation of reduced stress take the following form :

$$\sigma_{red} = \sqrt{\sigma^2 + 3\tau^2}$$

Energy methods

Castigliano's Theorem

generalized coordinates f_i corresponding to the force F_i is equal to the partial derivative of elastic energy V

$$f_i = \frac{\delta V}{\delta F_i}$$

Menabrei's Theorem

partial derivative of the elastic energy V of the system with respect to statically indeterminate reaction X is equal to zero

$$\frac{\delta V}{\delta X} = 0$$