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Fundamentals of mechanics and strength of materials

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Lecture program

1. The principles and basic concepts, fundamentals of vector account
2. Flat balance of forces - definitions, principles of reduction conditions of equilibrium, torque
3. Beams statically determinable - solving using analytical and graphical string polygon
4. Trusses flat statically determinable
5. Sliding and rolling friction
6. Static moment - measures weight solids and plane figures

Literature

1. **TAYLOR J., Classical mechanics,** University Science Books, 2005
2. **SCHECK F., Mechanics** - From Newton's Laws to Deterministic Chaos, Springer, 2003
3. **SINGH U.K., DWIVEDI M.,** Problems and solutions in mechanical engineering, New Age International, 2007
4. **AMBROSE J., Simplified mechanics and strength of materials,** New York , John Wiley & Sons, 2002.
5. **MISIAK J., Mechanika Ogólna,** WNT, Warszawa 1998 .
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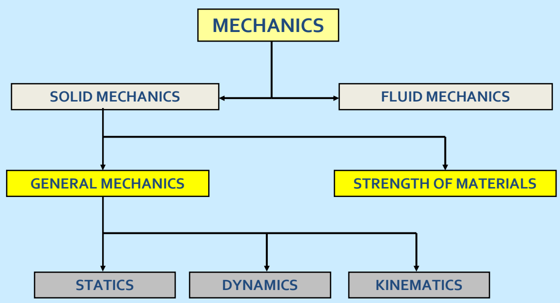
Division of mechanics



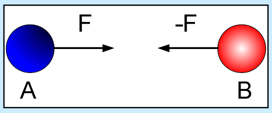
Newton's laws

I. Every body continues in its state of rest or of uniform rectilinear motion, except if it is compelled by forces acting on it to change that state.
The principle of inertia

II. The change of motion is proportional to the applied force and takes place in the direction of the straight line along which that force acts.

$$\vec{F} = \frac{d\vec{p}}{dt}$$

III. To every action there is always an equal and contrary reaction; or, the mutual actions of any two bodies are always equal and oppositely directed along the same straight line.



Newton's laws

The principle of superposition

If a material point of mass m runs at the same time several forces, each of which operates independently, and together they act as though there is only one force vector equals the vector sum of forces

The law of gravity

Any two material points attract each other with a force directly proportional to the product of their masses (m, M) and inversely proportional to the square of the distance r between them. The direction of force lies on the line joining these points

$$F = k\frac{m\,M}{R^2}$$

k – gravitational constant

Basic concepts

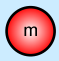
Basic concepts

- Mass, kg
- Force, N
  - Internal / External
  - Surface / Mass
  - Active / Reactive
- Space, m

Basic concepts

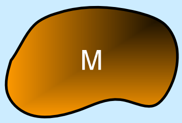
Material point

A body size vanishingly small compared with the size of the areas in which it moves. It is thus endowed with a certain geometric point mass



Perfectly rigid body

(indeformable), it is the body which points do not change within the forces



Basic concepts

System of material points

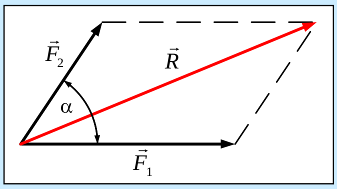
It arises from the breakdown of the body in the infinitely growing number of material points forming a continuum material.

Mechanical System

It is the set of material points or body having the property that the position and movement of each element depends on the position and movement of other elements of the system

The principle of static

The action of two forces $\vec{F}_1$ and $\vec{F}_2$ can be replaced by one force $\vec{R}$



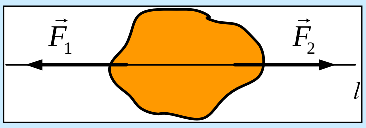
whose numerical value is :

$$R = \sqrt{\vec{F}_1 + \vec{F}_2 + 2\vec{F}_1\vec{F}_2 \cos\alpha}$$

The principle of statics

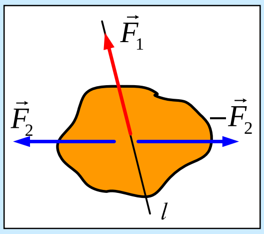
If applied to the body are two forces in balance, they are only when they have the same line of action, the same numeric value and returns the opposite

$$\vec{F}_1 = -\vec{F}_2 \qquad F_1 = F_2$$



The principle of statics

The effect of any system of forces applied to the body does not change if we add or subtract to this agreement an arbitrary system of balancing the forces of $\vec{F}_2$ and $-\vec{F}_2$. *Zero system*



Principle of statics

Principle of operation and prevent

Each activity is accompanied by equal in amount to the opposite return and lying on the same line counter

The principle of liberation from the constraints of any body can liberate from the bonds, replacing them with performance feedback, and then considering a free body, which is under the action of passive and active forces (reaction ties)

Principle of statics

Degree of freedom

The possibility of making the body motion is independent of other movements

Material point

Has two at the level, three in the space degree of freedom

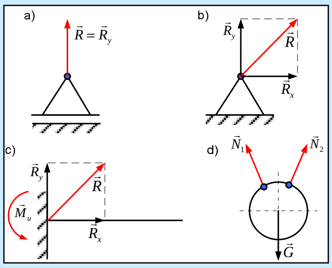
Perfectly rigid body

Has three at the level, six in the space degree of freedom

Principle of statics

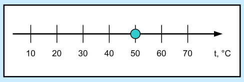
Bonds

conditions restricting the movement of the body in space



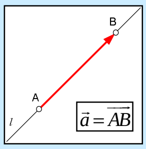
Physical quantities

Scalar



- temperature, °C
- density, kg/m³
- mass, kg
- volume, m³

Vector



$$\vec{a} = \left(\vec{a}_x, \vec{a}_y, \vec{a}_z\right)$$

$$\vec{a} = \left(B_x - A_x, B_y - A_y, B_z - A_z\right)$$

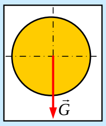
$$\vec{a} = a_x\vec{i} + a_y\vec{j} + \vec{a}_z$$

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

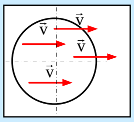
- velocity, m/s
- aceleration, m/s²
- force, N
- momentum, (kg m)/s

Types of vectors

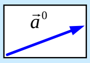
Related to point



Free

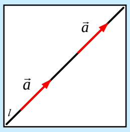


Wersor

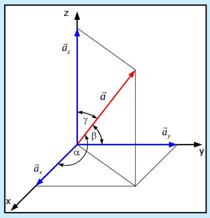


$$\left|\vec{a}^0\right| = a^0 = 1$$

Associated with straight line



Vector in a Cartesian's system



Directional cosines

$$\cos\alpha = \frac{a_x}{a} \qquad \cos\beta = \frac{a_y}{a} \qquad \cos\gamma = \frac{a_z}{a}$$

Operations on vectors

Addition → $\vec{c} = \vec{a} + \vec{b}$

Subtraction → $\vec{c} = \vec{a} - \vec{b}$

Scalar multiplication → $\vec{c} = d \cdot \vec{a}$

Vector multiplication → $\vec{c} = \vec{a} \times \vec{b}$, $d = \vec{a} \circ \vec{b}$

Systems of forces

The collection of any number of forces simultaneously acting on the body called the arrangement of force

Systems of forces are divided into:

Flat layouts

Spatial layouts

Flat systems are divided into:

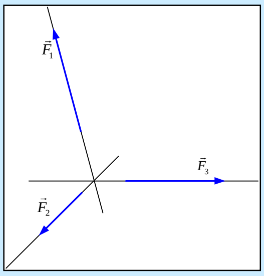
Flat converge systems

Plane parallel systems

Any flat layouts

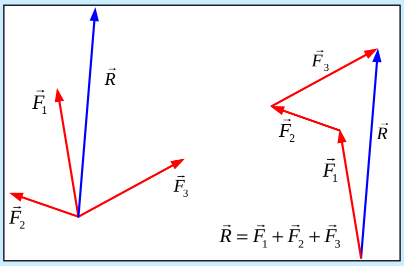
Convergent force system

Forces, whose lines of action intersect at one point called the Convergent force system



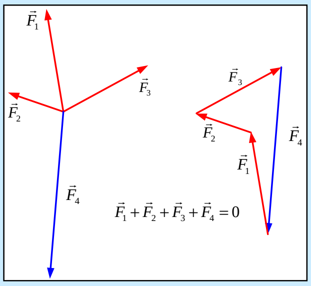
Reduction of converging forces

Resultant vector is anchored at the point of convergence of reducing system



The balance of converging forces

Convergent force system is in equilibrium if the polygon of forces of this system is closed. This is called graphical condition for the balance of converging forces.



The balance of converging forces

The analytical conditions of equilibrium of forces converging flat converging

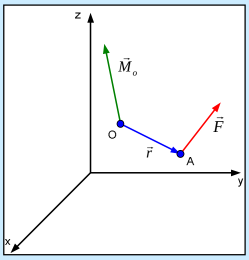
$$\sum_{i=1}^{n} F_{ix} = O$$

The sum of projections of all forces on the x-axis must be zero

$$\sum_{i=1}^{n} F_{iy} = O$$

The sum of projections of all forces on the y-axis must be zero

Moment of force with respect to the point



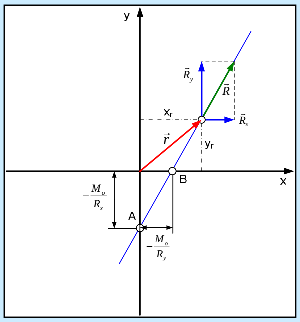
Moment of $\vec{F}$ force with respect to point o, the system x, y, z

$$\vec{M}_o = \vec{r} \times \vec{P} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r_x & r_y & r_z \\ P_x & P_y & P_z \end{vmatrix} =$$
$$= \vec{i}\left(r_y P_z - r_z P_y\right) + \vec{j}\left(r_z P_x - r_x P_z\right) + \vec{k}\left(r_x P_y - r_y P_x\right)$$

where:

$r_x = x_2 - x_1$ $\quad r_y = y_2 - y_1$ $\quad r_z = z_2 - z_1$

Moment of force with respect to the point



$$\vec{r} \times \vec{R} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_R & y_R & 0 \\ R_x & R_y & 0 \end{vmatrix} = \vec{k}\left(x_R R_y - y_R R_x\right) = \vec{M}_o =$$
$$= \vec{k} M_o = \sum_{i=1}^{n} \vec{M}_{io}$$

Thence

$$x_R R_y - y_R R_x = M_o$$

$$\frac{x_R}{\frac{M_o}{R_y}} - \frac{y_R}{\frac{M_o}{R_x}} = 1$$

Moment of force with respect to the point

In the system, four cases may occur

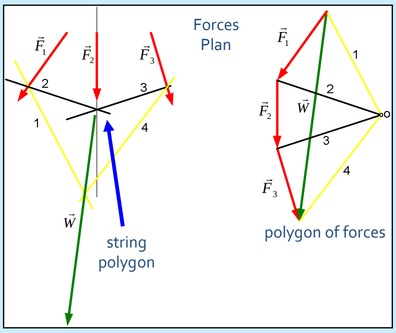
$\vec{R} \neq 0;\ \vec{M}_o \neq 0$ — system is reduced to the resultant force and torque of the main

$\vec{R} \neq 0;\ \vec{M}_o = 0$ — system is reduced to passing through the center of the resultant reduction o

$\vec{R} = 0;\ \vec{M}_o \neq 0$ — system is reduced to a pair of forces lying in the plane oxy

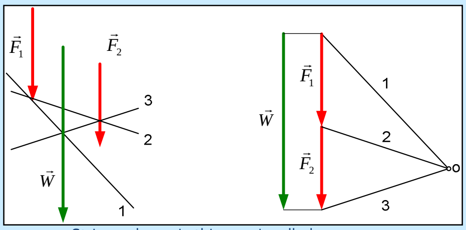
$\vec{R} = 0;\ \vec{M}_o = 0$ — **system is in equilibrium**

The reduction of forces by the string polygon



The method of the string polygon

Polygon of forces is open - the string polygon extreme rays intersect at one point. The system has a resultant force equal to the principal vector, passing through the intersection of the extreme rays of the string polygon.

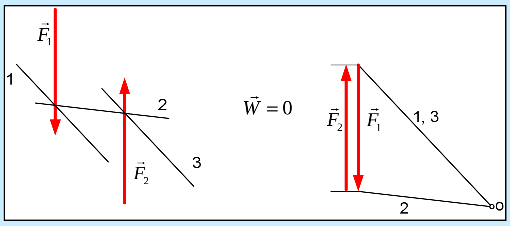


String polygon in this case is called open.

The method of the string polygon

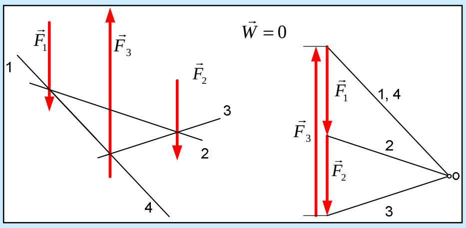
Polygon of forces is closed, the string polygon is open to the extreme rays of the string polygon are parallel to each other

The force system is reduced to a pair, which define the extreme rays of the string polygon



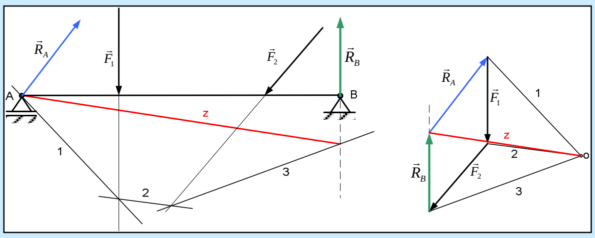
The method of the string polygon

Polygon of forces is closed, the extreme rays of the string polygon lie on a common line The system of forces is in equilibrium. Closed string polygon.



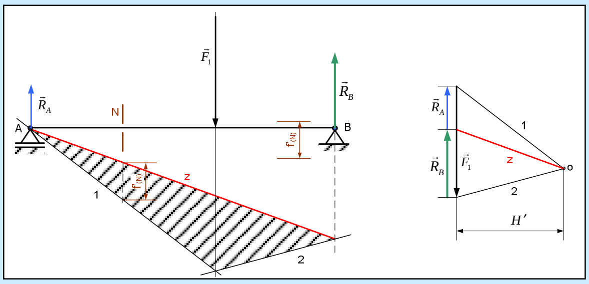
Determination of the reaction supports

Determination of the reaction supports the string polygon method



Determination of the distribution of bending moment

Bending moment in a cross-section of the body is the sum of the moments of all external forces acting on only one side of this section



Determination of the bending moment

To determine the value of the bending moment in cross-section N, the string polygon read ordinate $f'_{(N)}$ and calculate its value from the formula:

$$M_{(N)} = f'_{(N)} H' k_l k_p$$

where:

$M_{(N)}$ – bending moment in cross-section N

$f'_{(N)}$ – ordinate of the diagram of moments

H' – distance from the pole

k_l – length scale of the plan forces , np. m/cm,

k_p – Scale of a force , np. kN/cm.

Calculating reactions and moments in simple beams

Reactions of the supports in beams loaded flat arrangement of forces we determine the three equilibrium equations :

1. The sum of projections of all forces on the x-axis must be zero

$$\sum F_{ix} = 0$$

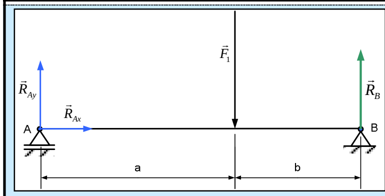
2. The sum of projections of all forces on the y-axis must be zero

$$\sum F_{iy} = 0$$

3. The sum of the moments of any chosen pole must be zero

$$\sum M_{iA} = 0$$

Calculating reactions and moments in simple beams



Equilibrium equations :

$$\sum F_{ix} = R_{Ax} = 0$$

$$\sum F_{iy} = R_{Ay} - F_1 + R_B = 0$$

$$\sum M_{iA} = -F_1 a + R_B(a+b) = 0$$

From the above equations shows that:

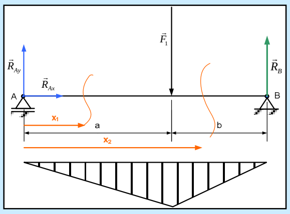
$$R_{Ax} = 0 \qquad R_{Ay} = F_1 \frac{b}{a+b} \qquad R_B = F_1 \frac{a}{a+b}$$

$$R_A = \sqrt{R_{Ax}^2 + R_{Ay}^2} = R_{Ay}$$

Calculating reactions and moments in simple beams

Distribution of bending moment equation describes :

$$\text{I. } 0 \le x_1 \le a$$
$$M(x_1) = R_{Ay} x_1 = F_1 \frac{b}{a+b} x_1$$
$$M(0) = 0, M(a) = F_1 \frac{b}{a+b} a$$



$$\text{II. } a \le x_2 \le a+b$$
$$M(x_2) = R_{Ay} x_2 - F_1(x_2 - a) = F_1 \frac{b}{a+b} x_2 - F_1(x_2 - a)$$
$$M(a) = F_1 \frac{b}{a+b} a, \; M(a+b) = 0$$

A statically determinable flat truss

Flat truss is called the system of rods lying in one plane, joints and connected in nodes.

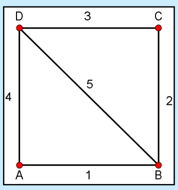
Flat trusses may be self-supporting structure, or be part of a larger structure - the spatial grid .

The actual load-bearing structures, such as the regimes maintaining floors of buildings, parts of cranes, etc., the rods are connected at nodes on a permanent basis (riveted, welded, bolted with bolts, etc.), but this does not significantly affect the calculation results .

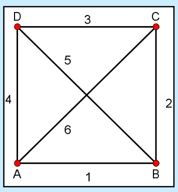
Characteristics of the truss

The real trusses are characterized by :

- the shape of the outer contour ,
- internal system of bars ,
- way of support (fixing) the grid ,
- as the load ,
- shape and dimensions of cross sections of bars



a) statically determinable



b) statically indeterminate

The calculation of statically determinate plane trusses

Assumptions:

- rods are pivotally connected and therefore are transferred only normal stresses: compressive or tensile ,

- rods are straight and weightless, and the forces applied only at nodes ,

- in statically determinate truss condition must be fulfilled p=2w-3.

Analytical method for nodes balancing

Example

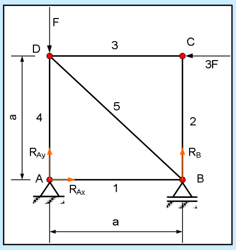
6 nodes i 9 rodes, so
p = 2w-3 = 2·6-3 = 9 is satisfied

Equilibrium conditions:

$$\sum F_{ix} = R_{Ax} - 3F = 0,$$

$$\sum F_{iy} = R_{Ay} + R_B - F = 0$$

$$\sum M_{iA} = -R_B a - 3Fa = 0$$



The flat truss

Reactions : $R_{Ax} = 3F$ $R_{Ay} = 4F$ $R_B = -3F$

Analytical method for nodes balancing

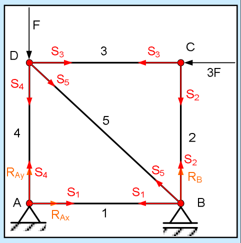
Now write the equilibrium conditions for the subsequent nodes

Equilibrium conditions for node A

$$\sum F_{ix} = R_{Ax} + S_1 = 0$$

$$\sum F_{iy} = R_{Ay} + S_4 = 0$$

$$S_1 = -R_{Ax} = -3F, \quad S_4 = -R_{Ay} = -4F$$



Equilibrium conditions for node D :

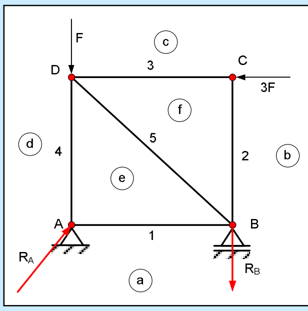
$$\sum F_{ix} = S_3 + S_5 \cos(45°) = 0, \quad \sum F_{iy} = -F - S_4 - S_5 \sin(45°) = 0$$

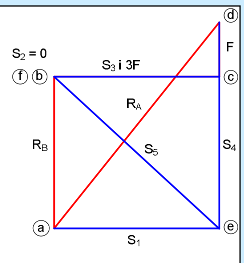
$$S_5 = 3\sqrt{2}F, \quad S_3 = -3F$$

Cremona's plan forces

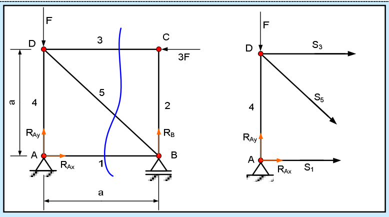
Cremona's plan forces is a diagram of a method involving the construction of polygon of forces for each node, starting from the node where the two rods are connected.

Cremona's plan forces





Analytical method of intersections (Ritter's method)



Reactions :

$R_{Ax} = 3F$

$R_{Ay} = 4F$

$R_B = -3F$

Equilibrium conditions:

$\sum F_{ix} = S_3 + R_{Ax} + S_1 + S_5 \cos(45°) = 0,$

$\sum F_{iy} = R_{Ay} - F - S_5 \sin(45°) = 0$

$\sum M_{iD} = -S_1 - R_{Ax} = 0$

$S_1 = -3F$

$S_5 = 3\sqrt{2}F$

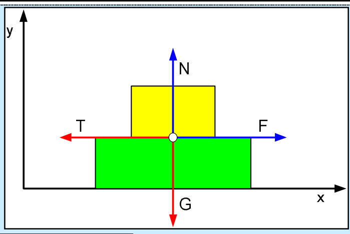
$S_3 = -3F$

Friction and friction law

Friction – phenomenon of the emergence of forces tangential to the contact surface of two bodies.

These are resistance forces. They prevent movement, which would made if there were no friction. They are therefore passive forces .

Coulomb's Experience



$\sum F_{ix} = F - T = 0$

$\sum F_{iy} = N - G = 0$

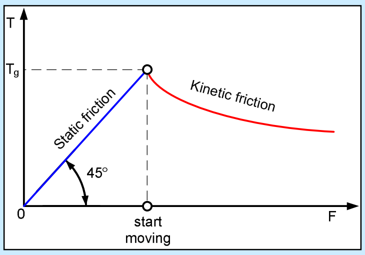
From that

$P = T$

$N = G$

Static and kinetic friction

The relationship between the friction force T and acting force F is shown below



Friction laws

1. The friction force is independent of surface area in contact with each other bodies and depends only on its type .
2. The size of the frictional forces of the body that is at rest can vary from zero to the limit, proportional to the total normal pressure.
3. When the body slides on a surface, friction force is always directed opposite to the direction of motion and is less than the limit value.

Friction force at rest

On the basis of these rights, you can specify the relationship between the friction force T and the pressure normal N. In the case of the body remaining at rest on the rough surface of the friction force dependence is expressed:

$$T \leq \mu N$$

where **μ** - coefficient of friction (static), depending on the type of friction material of the bodies, the values of the roughness and the surface (dry, wet, cold hot).

Boundary friction

If the friction force reaches its limit value, ie the friction is fully developed, the above formula should be an equal sign

$$T_g = \mu N$$

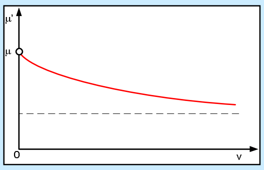
The direction of friction force T, acting on a body found at rest, is opposite to the direction of movement, which arose that if the friction was not .

Kinetic friction force

In the case of a body sliding on a rough surface the friction force is directed opposite to the direction of movement, and its value is given by

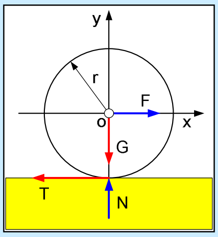
$$T' = \mu' N$$

where **μ'** - coefficient of sliding friction (kinetic), which depends on the relative speed of the body, according to the curve shown in Figure



Dependence of kinetic coefficient of friction on the relative velocity of the body v

Rolling friction



Equilibrium conditions

$$\sum F_{ix} = F - T = 0$$

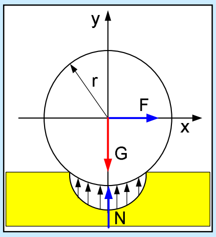
$$\sum F_{iy} = N - G = 0$$

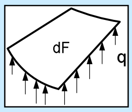
$$\sum M_{i0} = Tr = 0$$

$$T = F \qquad N = G \qquad T \neq 0$$

Rolling friction

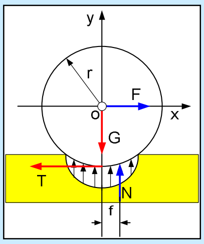
Rolling friction or rolling resistance is formed by shunting a cylinder with a weight of G on the horizontal plane.





$$N = \int q dF$$

Rolling friction



Equilibrium conditions

$$\sum F_{ix} = F - T = 0 \qquad T = F$$

$$\sum F_{iy} = N - G = 0 \qquad N = G$$

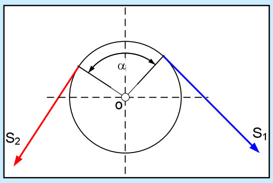
$$\sum M_{i0} = Tr - Nf = 0$$

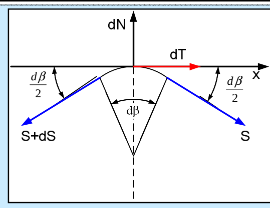
Substituting *N* = *G* we get:

$$\boldsymbol{T = G\frac{f}{r}}$$

f - rolling friction or rolling friction arm

Friction tension





$$\Sigma F_{ix} = -(S + dS)\cos\frac{d\beta}{2} + S\cos\frac{d\beta}{2} + dT = 0$$

$$\Sigma F_{iy} = dN - S\sin\frac{d\beta}{2} - (S + dS)\sin\frac{d\beta}{2} = 0$$

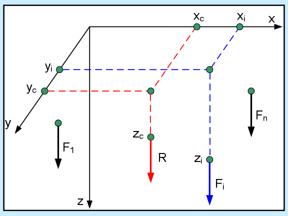
$$S_2 = S_1 e^{\mu\alpha}$$

Measure of parallel forces

Measure of parallel forces is called point C, through which runs the line of resultant force of the system

The coordinates of this measure, by definition, we determine the moment of force and allegations about the time of the resultant force .

Measure of parallel forces



The figure shows a balance of forces parallel to the axis and from the same structure rotated by an angle of 90 °.

The value of the y-axis until the resultant force R applied to point C is equal to

$$M_y = R\, x_c$$

Where in the resultant force module of this system is

$$R = \sum F_i$$

Measure of parallel forces

Moment, can also be written as

$$M_y = \sum F_i x_i$$

The theorem on the moment of collision there

$$R\, x_c = \sum F_i x_i$$

and hence

$$x_C = \frac{\sum_{i=1}^{i=n} F_i x_i}{R} = \frac{\sum_{i=1}^{i=n} F_i x_i}{\sum_{i=1}^{i=n} F_i}$$

Measure of parallel forces

Similarly, comparing the moments we obtain the y-axis and z-axis

$$y_C = \frac{\sum_{i=1}^{i=n} F_i y_i}{R} = \frac{\sum_{i=1}^{i=n} F_i y_i}{\sum_{i=1}^{i=n} F_i}$$

$$z_C = \frac{\sum_{i=1}^{i=n} F_i z_i}{R} = \frac{\sum_{i=1}^{i=n} F_i z_i}{\sum_{i=1}^{i=n} F_i}$$

Models of real objects

Material Line – body, whose mass is distributed along the line (wire rod). When the weight is distributed evenly on the line, we are dealing with a homogeneous body whose linear density is determined by the formula

$$\rho_l = \frac{m}{l} \qquad \frac{kg}{m}$$

where: *m* is the mass, is the length *l* of the rod

Models of real objects

Surface Material – two-dimensional solution, which replace the body with two dimensions are significantly higher than the third (sheet, plate, etc.). Homogeneous material for the surface density of the surface is defined as

$$\rho_S = \frac{m}{S} \qquad \frac{kg}{m^2}$$

where: *S* is the area

Models of real objects

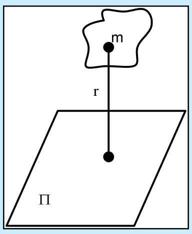
A lump of material – is the creation of three-dimensional. The density of a homogeneous solid material in the form we define

$$\rho = \frac{m}{V} \qquad \frac{kg}{m^3}$$

where: *V* is the volume of solids .

Static moment

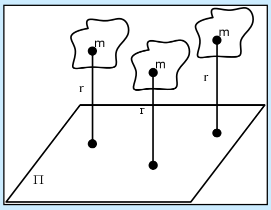
Static moment – material point relative to the plane Π is called the product of the mass of a point by its distance from this plane



$$S_\Pi = mr$$

Static moment

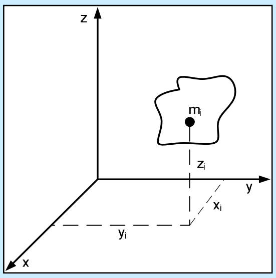
Static moment of system of material points plane Π is the sum of the static moments of all the points of this system, relative to this plane



$$S_\Pi = \sum S_{\Pi_i} = \sum m_i r_i$$

Static moment

In the rectangular coordinates system of material points is three moments of static models



$$S_{yz} = \sum m_i x_i$$

$$S_{zx} = \sum m_i y_i$$

$$S_{xy} = \sum m_i z_i$$

Static moments of the continuous system

$$S_{yz} = \int xdm \qquad S_{zx} = \int ydm \qquad S_{xy} = \int zdm$$

For a flat system :

$$S_y = \sum m_i x_i \qquad S_x = \sum m_i y_i$$

or

$$S_y = \int xdm \qquad S_x = \int ydm$$

Mass measure

For the **continuous system** equations takes the form :

$$mx_C = \int xdm \qquad my_C = \int ydm \qquad mz_C = \int zdm$$

Spatial center of mass has coordinates

$$x_C = \frac{\sum m_i x_i}{m} \qquad y_C = \frac{\sum m_i y_i}{m} \qquad z_C = \frac{\sum m_i z_i}{m}$$

Center of gravity of the solid material

Approximate formulas

$$x_C = \frac{\sum_{i=1}^{i=n} \gamma_i x_i \Delta V_i}{\sum_{i=1}^{i=n} \gamma_i \Delta V_i}$$

$$y_C = \frac{\sum_{i=1}^{i=n} \gamma_i y_i \Delta V_i}{\sum_{i=1}^{i=n} \gamma_i \Delta V_i}$$

$$z_C = \frac{\sum_{i=1}^{i=n} \gamma_i z_i \Delta V_i}{\sum_{i=1}^{i=n} \gamma_i \Delta V_i}$$

Center of gravity of the solid material

Precise patterns

$$x_C = \frac{\int_V \gamma x dV}{\int_V \gamma dV} = \frac{\int_V \gamma x dV}{G}$$

$$y_C = \frac{\int_V \gamma y dV}{\int_V \gamma dV} = \frac{\int_V \gamma y dV}{G}$$

$$z_C = \frac{\int_V \gamma z dV}{\int_V \gamma dV} = \frac{\int_V \gamma z dV}{G}$$